Solidification by Radiative Cooling of a Cylindrical Region Filled with Drops

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Transient cooling by radiation is analyzed for a cylindrical region filled with axially flowing streams of drops that are becoming solidified. This is of interest for the dissipation of waste heat from orbiting power systems in space. The drops absorb, emit, and scatter radiation, and the surroundings are at a lower uniform temperature. The radiative properties are assumed gray, and the scattering is isotropic. The radiating region is a two-phase mixture that remains at the melting temperature of the drops. Its temperature uniformity maintains a high emissive power as energy is lost. This is an advantage over a sensible heat radiator in which the temperature decreases, thereby reducing the emissive power. The results provide the axial length that remains two-phase and the fraction of energy dissipated within this length in which the emissive power has not decreased because of sensible cooling. It is also shown how the radial distribution of the axial velocity of the drops can be modified to increase this energy fraction.

Nomenclature

a = absorption coefficient of absorbing-scattering region, $E_a \pi R_d^2 N$
D_n = function defined in Eq. (3b)
E_a, E_s = efficiency factors for absorption and scattering by a single drop
g = function defined in Eq. (3d)
I = source function in absorbing-scattering region; $\tilde{I} = (\pi I/\sigma - T_e^4)/(T_f^4 - T_e^4)$
K_1, K_2 = integration kernels defined in Eq. (4b)
N = number of drops per unit volume
q_r = radiative heat flow per unit area and time
R = radial coordinate in cylinder; R_o, outer radius
R_d = radius of spherical drop
r = optical radius, $(a + \sigma_s)R$; $r_o = (a + \sigma_s)R_o$
r' = dummy variable for optical radius
T = absolute temperature
T_e = temperature of surrounding environment
T_f = solidification temperature
u = radial velocity distribution in cylindrical region
$\bar{u}$ = integrated mean value of u
$\tilde{v}$ = local liquid fraction of drops in region, V_ℓ/V_d
V_d = volume of drops per unit volume of region
V_ℓ = volume of liquid per unit volume of region
Z = axial coordinate along cylinder from drop origin
$\tilde{Z}$ = dimensionless length, $\sigma(T_f^4 - T_e^4)Z/\rho\lambda\bar{u}R_o$
$\tilde{Z}_o$ = axial location where $\tilde{V}$ first reaches zero
β = angle used in cylindrical geometry
$\Gamma^{\pm}$ = quantity defined in Eq. (3c)
ϵ_{ut} = emittance of cylinder at uniform temperature
K_D = optical thickness of a plane layer
λ = latent heat per unit mass of region containing drops
ρ = density of cylindrical region, $(4/3)\rho_d \pi R_d^3 N$
ρ_d = density of drop material
σ = Stefan-Boltzmann constant
σ_s = scattering coefficient in absorbing-scattering region, $E_s \pi R_d^2 N$
Ω = albedo for scattering, $\sigma_s/(a + \sigma_s)$

Introduction

THE liquid drop radiator currently is being investigated as a possible device for radiative dissipation of waste heat in space applications.[1-3] This is one of several types of advanced radiators that have been proposed for space use in an effort to significantly reduce size and weight as compared with conventional devices. In connection with evaluating the radiative performance of droplet-filled regions, the present author carried out some analyses of the transient cooling by sensible heat loss from plane and cylindrical regions that absorb, emit, and scatter radiation[4,5] in low-temperature surroundings. During transient cooling, the outer portion of the radiating region becomes cooler than its interior, and this lowers the emittance of the region. As the region cools, the local heat loss can be significantly reduced by the decrease in mean temperature. Since radiative loss depends approximately on the fourth power of the temperature, the heat dissipation is substantially reduced if only 20% of the original sensible energy is lost.

A method for eliminating the foregoing two effects that reduce radiative emissive power is to use a region filled with solidifying drops, as suggested in Ref. 6. As long as the entire region remains two-phase, it does not decrease either in outer temperature, thus maintaining a high emittance, or in mean temperature, thus maintaining its original emissive power. A shorter axial length thus would be required to dissipate the same amount of energy as a radiator using sensible heat. If the drops are collected before the region becomes completely solidified, there will be a mixture of solid and liquid. The liquid will provide good heat-transfer contact to the solid portion, thereby enabling the solid to be readily melted for reuse.

An additional technique is to modify the distribution across the region of the axial droplet velocity. By having larger droplet velocities in the outermost regions, more latent energy will be supplied, and the axial location at which complete solidification first occurs can be extended. This will increase the amount of latent energy that can be dissipated before the

Received May 10, 1988; revision received Aug. 8, 1988.

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outer region becomes solid and then sustains a temperature decrease as a result of sensible heat loss.

For a uniform velocity distribution across the cylinder, results are given for the axial location at which complete solidification first occurs locally within the region. The fraction of the latent heat that has been dissipated prior to this location is given, and it is compared with that of a plane layer as a function of both optical thickness and scattering albedo. The velocity distribution that will maintain a uniform liquid volume fraction during solidification within the region at each axial location is given. This distribution will maintain the region at uniform temperature until all the latent energy is radiated away. The analysis also includes incident energy from the external environment, which can have any uniform effective radiating temperature below the solidification temperature.

Analysis

The drops in the cylindrical region shown in Fig. 1 are formed from liquid slightly above its melting point, and any small superheat is neglected in the analysis. As the droplets solidify, it is assumed that a two-phase mixture exists until all of the latent energy is lost from within a local region. When the drops all have the same axial velocity, complete solidification first occurs at the outer boundary of the cylinder. Until this condition is reached, the entire cross section remains at T_f, and the radiative cooling is governed by the energy equation in terms of the radiative flux,

$$\frac{\rho\lambda u(R)}{V_d}\frac{\partial V_\ell}{\partial Z} = -\frac{1}{R}\frac{\partial}{\partial R}[Rq_r(R)] = -\frac{a+\sigma_s}{r}\frac{\partial}{\partial r}[rq_r(r)] \quad (1)$$

Since the temperatures remain constant in the axial direction for the two-phase mixture and for the surroundings, the gradient of the axial radiative flux is neglected compared with that in the radial direction. The radial flux gradient is much larger because the surroundings are at a lower temperature T_e. From Ref. 5, the radiative source function for emission and scattering can also be written in terms of $q_r(r)$ as

$$I = \frac{\sigma T_f^4}{\pi} - \frac{\Omega}{1-\Omega}\frac{1}{4\pi}\frac{1}{r}\frac{\partial}{\partial r}[rq_r(r)] \quad (2)$$

The radiative properties are assumed gray, and the scattering is isotropic. These assumptions are discussed in Ref. 5. For uniform radiative properties, the radiative flux can be written,[5,7] where it is assumed that the surroundings act like a black background at T_e, as

$$q_r(r) = 4\frac{r_o}{r}\left\{\int_{r'=0}^{r} I(r')\left[\int_{\beta=0}^{\sin^{-1}(r'/r_o)} \frac{D_2[\Gamma^-(r,r',\beta)] + D_2[\Gamma^+(r,r',\beta)]}{g(r',\beta)} \times \cos\beta\, d\beta\right]dr' + \int_{r'=r}^{r_o} I(r')\left[\int_{\beta=0}^{\sin^{-1}(r/r_o)} \frac{-D_2[\Gamma^-(r',r,\beta)] + D_2[\Gamma^+(r',r,\beta)]}{g(r',\beta)} \times \cos\beta\, d\beta\right]dr'\right\} + \frac{\sigma T_e^4}{\pi}\int_{\beta=0}^{\sin^{-1}(r/r_o)} \{D_3[2\Gamma^-(r,r_o\sin\beta,\beta) + \Gamma^-(r_o,r,\beta)] - D_3[\Gamma^-(r_o,r,\beta)]\}\cos\beta\, d\beta \quad (3a)$$

where

$$D_n(\xi) = \int_0^{\pi/2}(\cos\alpha)^{n-1}e^{-\xi/\cos\alpha}d\alpha = \int_0^1 \frac{\mu^{n-1}e^{-\xi/\mu}}{(1-\mu^2)^{1/2}}d\mu \quad (3b)$$

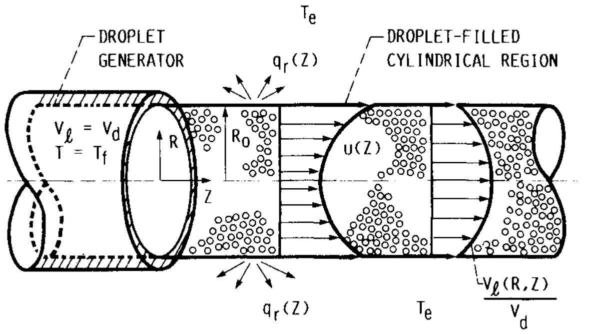

Fig. 1 Cylindrical stream of solidifying drops that absorb, emit, and scatter radiation.

$$\Gamma^{\pm}(b,c,\beta) = [b^2 - (r_o\sin\beta)^2]^{1/2} \pm [c^2 - (r_o\sin\beta)^2]^{1/2} \quad (3c)$$

$$g(r',\beta) = \left[1 - \left(\frac{r_o}{r'}\right)^2\sin^2\beta\right]^{1/2} \quad (3d)$$

For use in the energy equation, the differentiation of $rq_r(r)$ yields

$$-\frac{1}{r}\frac{\partial}{\partial r}[rq_r(r)] = -4\pi\left[I(r) - \int_{r'=0}^{r} I(r')K_1(r',r)\,dr' - \int_{r'=r}^{r_o} I(r')K_2(r',r)dr'\right] + 4\frac{\sigma T_e^4}{\pi}\frac{r_o}{r}\int_{\beta=0}^{\sin^{-1}(r/r_o)} \frac{D_2[\Gamma^+(r_o,r,\beta)] + D_2[\Gamma^-(r_o,r,\beta)]}{g(r,\beta)}\cos\beta\, d\beta \quad (4a)$$

where

$$K_1(b,c) = \frac{b}{c}K_2(c,b) = \frac{1}{\pi}\frac{r_o}{c}\int_{\beta=0}^{\sin^{-1}(b/r_o)} \frac{D_1[\Gamma^-(c,b,\beta)] + D_1[\Gamma^+(c,b,\beta)]}{g(c,\beta)g(b,\beta)}\cos\beta\, d\beta \quad (4b)$$

It is now noted that

$$\int_0^r K_1(r',r)\,dr' = \frac{1}{\pi}\frac{r_o}{r}\int_0^{\sin^{-1}(r/r_o)} \frac{1 - D_2[\Gamma^+(r,r,\beta)]}{g(r,\beta)}\cos\beta\, d\beta \quad (5a)$$

$$\int_r^{r_o} K_2(r',r)\,dr' = \frac{1}{\pi}\frac{r_o}{r}\int_0^{\sin^{-1}(r/r_o)} \frac{1 - D_2[\Gamma^-(r_o,r,\beta)] - D_2[\Gamma^+(r,r_o,\beta)] + D_2[\Gamma^+(r,r,\beta)]}{g(r,\beta)} \times \cos\beta\, d\beta \quad (5b)$$

and

$$\frac{r_o}{\pi r}\int_0^{\sin^{-1}(r/r_o)} \frac{2}{g(r,\beta)}\cos\beta\, d\beta = 1 \quad (5c)$$

With these relations, the last integral in Eq. (3a) can be expressed in terms of K_1 and K_2, and the derivative of $rq_r(r)$

becomes

$$-\frac{1}{4\pi}\frac{1}{r}\frac{\partial}{\partial r}[rq_r(r)] = -\left[I(r) - \frac{\sigma T_e^4}{\pi}\right] + \int_0^r \left[I(r') - \frac{\sigma T_e^4}{\pi}\right] K_1(r',r)\,dr' + \int_r^{r_o}\left[I(r') - \frac{\sigma T_e^4}{\pi}\right]K_2(r',r)\,dr' \tag{6}$$

Equations (1), (2), and (6) are now combined, and $q_r(r)$ is eliminated. By using the dimensionless groups in the nomenclature to absorb the T_e^4 terms, the following equations are obtained for the radiative source function and the liquid fraction in the droplet stream:

$$\tilde{I}(r) = 1 - \Omega + \Omega\left\{\int_0^r \tilde{I}(r')K_1(r',r)\,dr' + \int_r^{r_o}\tilde{I}(r')K_2(r',r)\,dr'\right\} \tag{7}$$

$$\frac{\partial \tilde{V}(r,\tilde{Z})}{\partial \tilde{Z}} = -4r_o\frac{1-\Omega}{\Omega}\frac{1-\tilde{I}(r)}{u(r)/\bar{u}} \qquad 0<\Omega<1 \tag{8}$$

From Eq. (7), the $\tilde{I}(r)$ is independent of $\tilde{Z}$ and is a function of the two parameters Ω and r_o. Since at $\tilde{Z} = 0$, $\tilde{V}(r,0) = 1$, Eq. (8) can be integrated to yield

$$\tilde{V}(r,\tilde{Z}) = 1 - 4r_o\frac{1-\Omega}{\Omega}\frac{1-\tilde{I}(r)}{u(r)/\bar{u}} \qquad 0<\Omega<1 \tag{9}$$

For $\Omega = 0$, Eq. (7) shows that $\tilde{I}(r) = 1$, and Eq. (8) becomes indeterminate. However, if the quantity $1 - \tilde{I}(r)$ is substituted from Eq. (7) into Eq. (8), the Ω is eliminated. Then $\tilde{I}(r)$ is set equal to unity and the results integrated with respect to $\tilde{Z}$ to obtain

$$\tilde{V}(r,\tilde{Z}) = 1 - 4r_o \times \frac{1 - \left[\int_0^r K_1(r',r)\,dr' + \int_r^{r_o} K_2(r',r)\,dr'\right]}{u(r)/\bar{u}} \qquad \Omega = 0 \tag{10}$$

Equations (9) and (10) are valid until $\tilde{V} = 0$ at any r value; after that, the cylinder will no longer be entirely two-phase, and the use of $T = T_f$ in the energy relations is no longer valid. If the velocity $u(r)$ is uniform with r, then with increasing $\tilde{Z}$, the $\tilde{V}$ first becomes zero at $r = r_o$; the corresponding $\tilde{Z}$ at $\tilde{V} = 0$ is

$$\tilde{Z}_o = \frac{1}{4r_o}\frac{\Omega}{1-\Omega}\frac{1}{1-\tilde{I}(r_o)} \qquad 0<\Omega<1 \tag{11}$$

$$\tilde{Z}_o = \frac{1}{4r_o}\frac{1}{1 - \int_0^{r_o} K_1(r',r_o)\,dr'} \qquad \Omega = 0 \tag{12a}$$

From the integral of K_1 in Eq. (5a), this can be further reduced to

$$\tilde{Z}_o = \frac{1}{4r_o}\frac{1}{\frac{1}{2} + \frac{1}{\pi}\int_0^{\pi/2} D_2(2r_o\cos\beta)\,d\beta} \qquad \Omega = 0 \tag{12b}$$

which was evaluated numerically.

A situation of interest is the control of the radiative cooling by adjusting the velocity distribution $u(r)$ of the drops. A potentially useful case is to make the liquid fraction $\tilde{V}$ independent of r. Then, from Eq. (9), $u(r)$ must be proportional to $1 - \tilde{I}(r)$, and from the definition of $\bar{u}$,

$$\frac{u(r)}{\bar{u}} = \frac{1 - \tilde{I}(r)}{\frac{2}{r_o^2}\int_0^{r_o}[1 - \tilde{I}(r)]r\,dr} \qquad 0<\Omega<1 \tag{13}$$

For $\Omega = 0$, Eq. (10) yields

$$\frac{u(r)}{\bar{u}} = \frac{1 - \int_0^r K_1(r',r)\,dr' - \int_r^{r_o} K_2(r',r)\,dr'}{\frac{2}{r_o^2}\int_0^{r_o}\left[1 - \int_0^r K_1(r',r)\,dr' - \int_r^{r_o} K_2(r',r)\,dr'\right] r\,dr} \tag{14}$$

This was evaluated using the expressions in Eqs. (5a) and (5b) for the integrals of K_1 and K_2.

For the velocity distributions in Eqs. (13) and (14), the V_ℓ stays uniform at each Z. Hence, the entire cylinder will become solid at Z_o and the cylinder will have radiated away its entire latent energy. The Z_o is found from the energy balance $2\pi R_o Z_o \epsilon_{ut}\sigma(T_f^4 - T_e^4) = \pi R_o^2 \rho\lambda\bar{u}$, where $\pi R_o^2 \rho\bar{u}$ is the total liquid mass flow rate. This yields the dimensionless form

$$\tilde{Z}_o = \frac{1}{2\epsilon_{ut}} \tag{15}$$

The ϵ_{ut} are the emittances for a cylinder at uniform temperature and were computed in Ref. 5 as a function of r_o and Ω. A slightly augmented listing of values is found in Table 1.

For the situation where $u(r)$ is uniform across the cylinder, the ϵ_{ut} values can be used to obtain the fraction of energy that can be radiated away before complete solidification occurs at the outer boundary of the cylinder. This is given by

$$\frac{2\pi R_o \epsilon_{ut}\sigma[T_f^4 - T_e^4]Z_o}{\pi R_o^2 \bar{u}\rho\lambda} = 2\epsilon_{ut}(r_o,\Omega)\tilde{Z}_o(r_o,\Omega) \tag{16}$$

A convenient result for ϵ_{ut} can be obtained in the optically thin limit. For small r_o, Eq. (7) shows that $\tilde{I}(r)$ approaches the value $1 - \Omega$. This is inserted into Eq. (3a), which is evaluated at r_o. After simplification this becomes

$$q_r(r_o) = \sigma[T_f^4 - T_e^4](1-\Omega)\left[1 - \frac{4}{\pi}\int_0^{\pi/2} D_3(2r_o\cos\beta)\cos\beta\,d\beta\right]$$

From Ref. 8, for small r_o, $D_3(2r_o\cos\beta) = \pi/4 - 2r_o\cos\beta + \ldots$. Then

$$\epsilon_{ut} = \frac{q_r(r_o)}{\sigma[T_f^4 - T_e^4]} = (1-\Omega)2r_o \tag{17}$$

This confirms the trends of the values in Table 1 as r_o becomes small for each Ω.

Table 1 Emittance for cylinder at uniform temperature ϵ_{ut}

Optical radius r_o	Scattering albedo Ω						
	0	0.3	0.6	0.8	0.9	0.95	0.98
0.2	0.317	0.237	0.145	0.076	0.039	0.020	0.008
0.5	0.596	0.474	0.314	0.176	0.094	0.048	0.020
1.0	0.814	0.689	0.500	0.306	0.173	0.092	0.039
2.0	0.946	0.841	0.670	0.465	0.292	0.168	0.074
3.0	0.977	0.882	0.729	0.539	0.364	0.224	0.105
4.0	0.987	0.897	0.752	0.574	0.406	0.263	0.130
5.0	0.992	0.904	0.764	0.593	0.432	0.291	0.151
6.0	0.994	0.907	0.769	0.602	0.446	0.309	0.167
7.0	0.995	0.909	0.772	0.607	0.454	0.321	0.180

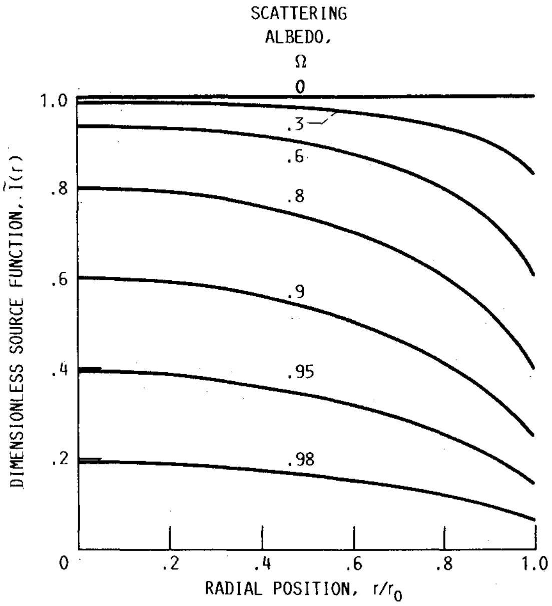

Fig. 2 Dimensionless source function $\tilde{I}(r)$ as a function of Ω for $r_o = 3$ and a uniform velocity distribution.

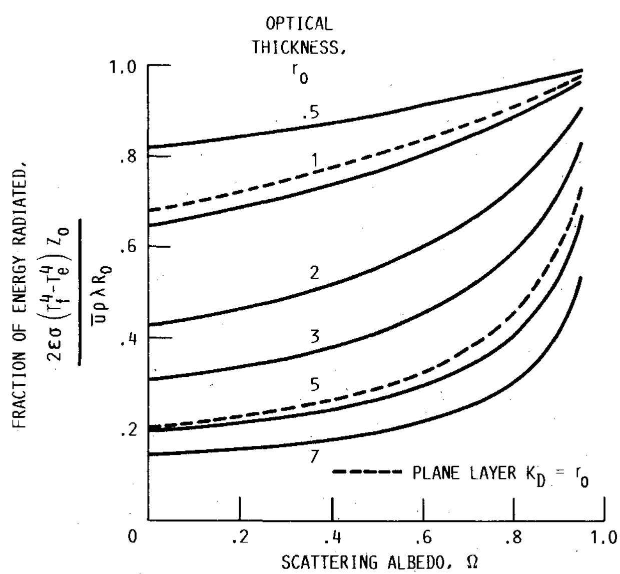

Fig. 3 Fraction of latent energy radiated away before any local region becomes completely solidified; $\epsilon = \epsilon_{ut}$ and values are in Table 1.

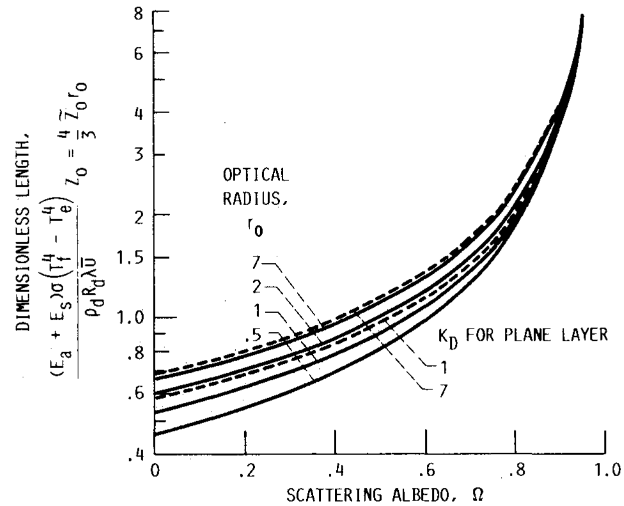

Fig. 4 Axial length in which entire cross section remains only partially solidified.

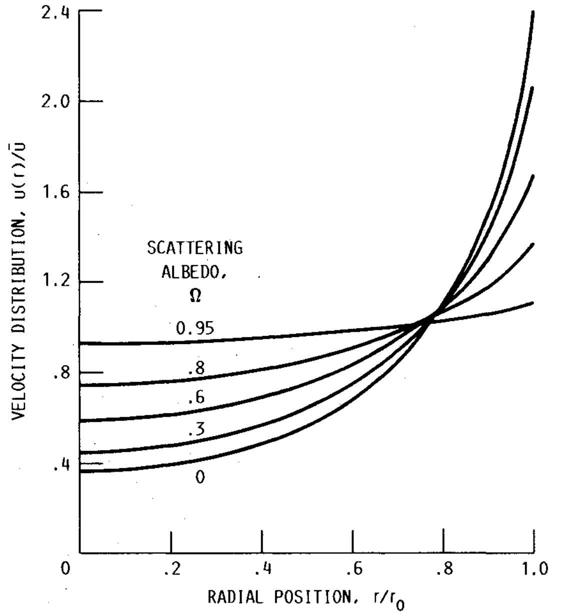

Fig. 5 Effect of albedo on velocity distribution required to maintain a uniform liquid fraction; $r_o = 2$.

The evaluation of results from Eqs. (9) and (11) requires $\tilde{I}(r)$, which was obtained numerically by iteration of Eq. (7). To begin the iteration, a constant value of $\tilde{I}$ was substituted into the right side. Carrying out the integration yields a new distribution for $\tilde{I}(r)$. The old $\tilde{I}(r)$ was subtracted from the new distribution and the difference multiplied by an acceleration factor of 1.2. The result was added to the old $\tilde{I}(r)$ to obtain a new approximation, which was substituted into the right side of Eq. (7) to continue the iteration. The characteristics of the K_1 and K_2 functions are given in Ref. 5, along with the integration procedure. Typical values of $\tilde{I}(r)$ are given in Fig. 2 for $r_o = 3$ and for various Ω.

Results and Discussion

The drop-filled region originates as liquid at a temperature slightly above the melting temperature T_f. For the solution given here, the radiating region is a two-phase mixture, and hence it remains at T_f. Radiative heat loss occurs from all locations within the region, and it is influenced by the amount of scattering present. The fact that the local radiative loss varies over the regional cross section creates a distribution in the liquid fraction. This is given by Eqs. (9) and (10). The functions containing the radial shape multiply $\tilde{Z}$, so that the $\tilde{V}$ decreases linearly with $\tilde{Z}$ while maintaining a similar shape with r. This is a consequence of the region remaining at uniform temperature while its latent energy is being dissipated, so that the radiative source function is only a function of r. As $\tilde{Z}$ increases, the $\tilde{V}$ eventually will decrease to zero somewhere within the region. Then, for larger $\tilde{Z}$, this completely solidified portion will decrease in temperature as sensible heat is lost. The present solution examines the radiative heat loss while the entire region remains two-phase and isothermal, thus retaining its high emissive power.

When the velocity distribution across the cylinder is uniform, the first drops to become completely solid are at the outer edge r_o. This occurs at the axial location $\tilde{Z}_o$ given by Eqs. (11) and (12). The fraction of the latent energy that has been radiated away between $\tilde{Z} = 0$ and $\tilde{Z}_o$ is given by Eq. (16), with the required ϵ_{ut} values in Table 1. The results are given in Fig. 3 as a function of optical outer radius and scattering albedo. The largest emitted fractions are for optically thin regions such as $r_o = 0.5$, as the radiative cooling is more uniform throughout the region, leading to a more uniform liquid fraction with radius. When the optical radius is increased to 2, at least 40% of the latent energy can be dissipated before the drops become solid at the outer radius. The portion dissipated increases as Ω is increased, because the scattering helps equalize the energy transfer across the layer. Thus, if the droplet material is somewhat reflective, a substantial amount of energy can be radiated away while maintaining the high emissive power $\epsilon_{ut}\sigma T_f^4$. For a liquid drop radiator operating with sensible heat loss only, the emissive power can decrease substantially as the droplet cloud temperature decreases. Thus, for sensible heat, a greater radiator length would be required to dissipate the same energy as the radiator using latent energy.

Figure 3 also shows a comparison of the cylindrical and plane-layer behavior for two optical thicknesses. The analysis in Ref. 6 was first generalized to account for nonzero T_e in the same manner as in the present analysis. The results for mean beam length[9] for an absorbing-emitting medium ($\Omega = 0$) indicate that the radiative flux from a plane layer with entire optical thickness K_D is approximately equal to that for a cylinder having an optical radius $r_o = K_D$. The comparisons in Fig. 3 show that this is also reasonably valid for any value of the scattering albedo.

The dimensionless axial position where complete local solidification is first achieved is given in Fig. 4, as computed from Eqs. (11) and (12). The ordinate was chosen to depend only on the droplet properties. As Ω is increased, the emissive ability of the layer decreases, and the required length increases. For high-temperature applications, it may be necessary to use liquid metals to avoid evaporation losses, and it may not be possible to have a small value of Ω. The comparisons with results for a plane layer reveal again that comparable behavior is obtained when the optical radius of the cylinder is equal to the entire optical thickness of the plane layer.

Figures 5 and 6 show the velocity distributions that will maintain a uniform liquid fraction across the cylinder at each axial position. The liquid fraction then decreases linearly with $\tilde{Z}$, and all of the drops becomes solid at the $\tilde{Z}_o$ value given by Eq. (15). The required velocity distributions are obtained from Eqs. (13) and (14). Figure 5 shows results for a fixed optical radius, $r_o = 2$, with various values of the scattering albedo. For $\Omega = 0$, the velocity would need to vary considerably with the radial coordinate. Large $u(r)$ values are needed near r_o to overcome the effect of large radiative heat loss in the vicinity of the cylinder exterior. As the albedo is increased, however, the required velocity distribution would not have such a large variation with radius and would be more practical to achieve. Figure 6 illustrates the effect of optical radius with the albedo kept fixed at $\Omega = 0.6$. The required velocity variations become more extreme as r_o increases. Additional trends for the velocity variations can be found from the results for a plane layer in Ref. 6.

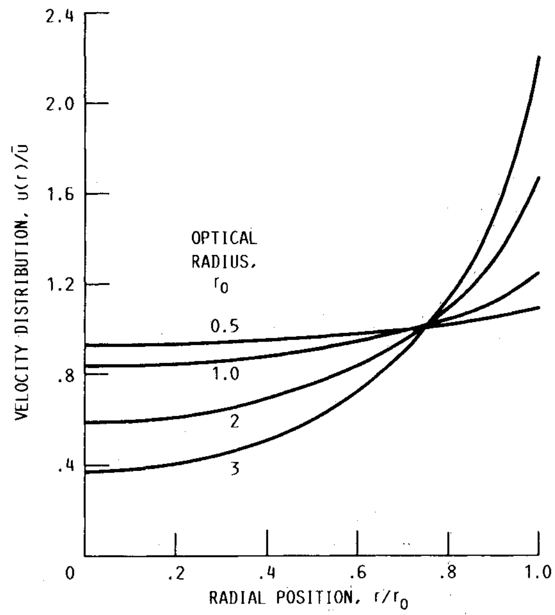

Fig. 6 Effect of optical radius on velocity distribution that will maintain a uniform liquid fraction; $\Omega = 0.6$.

Conclusions

The radiative heat dissipation was analyzed for a cylindrical stream of drops that emit, absorb, and scatter radiation. The drops originate at their melting temperature, and the droplet cloud partially solidifies before the drops are collected for remelting and reuse of the liquid. During the radiative dissipation, the region is two-phase and isothermal. This maintains a high radiative emissive power, since the emittance is not decreased by the outer edges of the region becoming cool, and the fourth power of the radiating temperature is not decreasing. The radiative behavior is governed by two parameters, the optical outer radius of the region and the scattering albedo. The fraction of the latent energy that can be dissipated before any part of the region becomes completely solidified is given. This is compared with a plane layer, and the results are similar if the cylinder optical radius equals the entire optical thickness of the plane layer. It is shown how the axial velocity distribution can be modified to make the liquid fraction independent of radius. This extends the length of the two-phase region and takes ad- vantage of its favorable radiating characteristics.

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